

YEAR 9 | NUMBER AND ALGEBRA

VOL.2

EQUATIONS &  
LINEAR RELATIONSHIPS

YEAR

9

MATHS

MAX SERIES™



THE HSC EXPERTS

### 03 CONSTRUCTING EQUATIONS

#### Using equations to solve word problems

- Word problems can be categorised into:
  - (i) Number Problems
  - (ii) Measurement Problems
  - (iii) Money Problems
  - (iv) Age Problems
- To solve word problems:
  - Step 1:** Underline the keywords that suggest mathematical operations.
  - Step 2:** Select a pronumeral to represent the unknown quantity.
  - Step 3:** Translate the word problem into an equation.
  - Step 4:** Solve the equation.
  - Step 5:** Check that the solution satisfies the given problem.
  - Step 6:** State your answer in words.
- The table below shows keywords for addition and subtraction.

Word	Operation	Word	Operation
Plus	+	Minus	-
Sum	+	Difference	-
More than	+	Less than	-
Increased by	+	Decreased by	-
Gain	+	Loss	-

- The table below shows keywords for multiplication and division.

Word	Operation	Word	Operation
Times	×	Ratio	÷
Multiplied by	×	Divided by	÷
Product of	×	Quotient of	÷
Twice or double	×2	Half	÷2
Triple	×3	One-third	÷3

- In measurement and money problems, don't forget to include units in your answer.

#### Number problems

- In number problems, you are required to find an unknown number given its relationship with the other number(s).
- Number problems could involve
  - (i) Two numbers
  - (ii) Two or three consecutive numbers
  - (iii) Two of three consecutive even or odd numbers

#### Example 1

The sum of two consecutive numbers is 87 .  
Find the numbers.

#### Solution

**Step 1:** Underline the keywords that suggest mathematical operations.

**Step 2:** Select a pronumeral to represent the unknown quantity.

Let the two consecutive numbers be  $x$  and  $x + 1$ .

**Step 3:** Translate the word problem into an equation.

The sum of two consecutive numbers = 87  

$$x + (x + 1) = 87$$

**Step 4:** Solve the equation.

$$\begin{aligned} x + (x + 1) &= 87 \\ 2x + 1 &= 87 \\ 2x &= 86 \\ x &= 43 \end{aligned}$$

**Step 5:** Check that the solution satisfies the given problem.

$$\begin{aligned} x &= 43 \\ x + 1 &= 43 + 1 = 44 \\ 43 + 44 &= 87 \end{aligned}$$

**Step 6:** State your answer in words.

The two consecutive numbers are 43 and 44.

#### Example 2

The sum of two consecutive even numbers is 146 .  
Find the numbers.

#### Solution

**Step 1:** Underline the keywords that suggest mathematical operations.

**Step 2:** Select a pronumeral to represent the unknown quantity.

Let the two consecutive even numbers be  $x$  and  $x + 2$ .

**Step 3:** Translate the word problem into an equation.

The sum of two consecutive even numbers = 146  

$$x + (x + 2) = 146$$

**Step 4:** Solve the equation.

$$\begin{aligned} 2x + 2 &= 146 \\ 2x &= 144 \\ x &= 72 \end{aligned}$$

**Step 5:** Check that the solution satisfies the given problem.

$$\begin{aligned} \text{The smaller even number: } x &= 72 \\ \text{The next consecutive even number: } x + 2 &= 74 . \end{aligned}$$

**Step 6 :** State your answer in words.

The two consecutive numbers are 72 and 74 .

## 03 CONSTRUCTING EQUATIONS (MEASUREMENT PROBLEMS)

11. A water storage tank is two-thirds full. After 500 litres of water is drained out, the tank is two-fifths full.
- (a) Let the full capacity of the water storage tank be  $x$  litres. Write an expression in  $x$  for the initial amount of water in the tank.
- (b) Write an equation and solve it to find the capacity of the water storage tank.

12. A truck is loaded with 80 drums. Each drum weighs either 45 kg or 30 kg. The total mass of the 80 drums is 2970 kg.
- (a) Let the number of drums weighing 45kg be  $x$ . Write an expression in  $x$  for the total mass of 45kg drums and for the total mass of 30 kg drums.
- (b) Form an equation and solve it to find the number of drums weighing 45 kg.

LEVEL

4  
3  
2  
1

NOTES TO STUDENTS

The amount of water in the tank before and after draining will be a fraction of  $x$ .

*Total weight =*  
*number of drums  $\times$  weight of drums*

The question only asks for the number of 45kg drums ( $x$ ).

# CONTENTS

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## EQUATIONS

### 01 SOLVING LINEAR EQUATIONS

P07

- What is a linear equation?
- What does it mean to solve a linear equation?
- Procedure for solving equations
- Equations with one pronumeral on one side
- Equations with pronumerals on both sides
- Equations with brackets
- Equations involving fractions

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### 02 REARRANGING EQUATIONS

P18

- Rearranging formula

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### 03 CONSTRUCTING EQUATIONS

P22

- Using equations to solve word problems
- Number problems
- Measurement problems
- Money Problems
- Age Problems

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### 04 INEQUALITIES

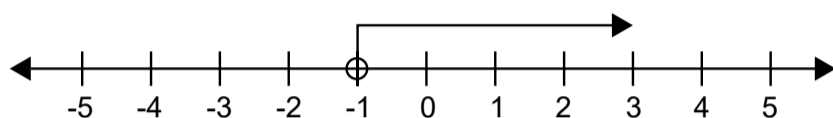
P35

- What is an inequality?
  - Graphing inequalities on the number line
  - Solving linear inequalities
  - Solving and graphing inequalities of the form  $a \leq x \leq b$
  - Solving and graphing inequalities involving fractions
-

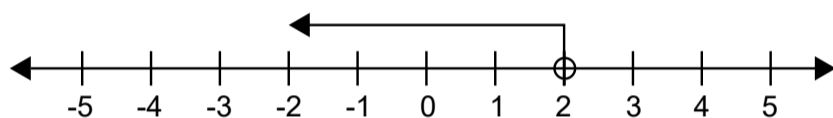
04 INEQUALITIES ( REPRESENTING LINEAR INEQUALITIES )

2. Write inequality statements expressed by the following number lines.

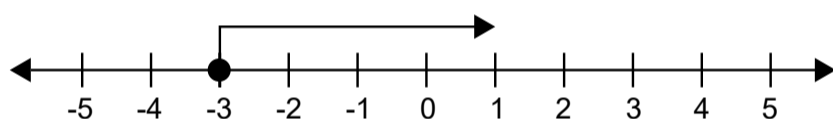
(a)



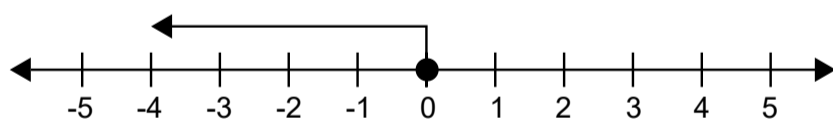
(b)



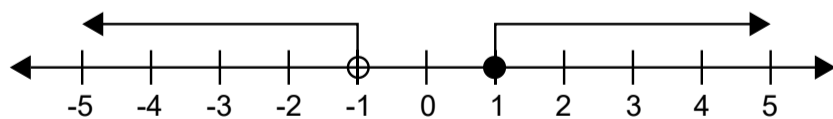
(c)



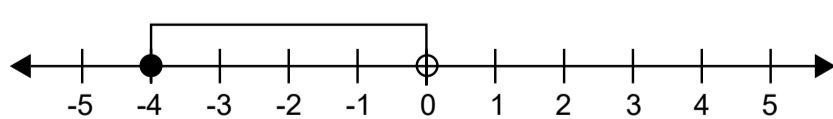
(d)



(e)



(f)



LEVEL

4  
3  
2  
1

NOTES TO STUDENTS

Use the arrow head to help you remember which inequality sign to use.

An arrow pointing to the right  $\rightarrow$  means  $>$ .

An arrow pointing to the left  $\leftarrow$  means  $<$ .

4  
3  
2  
1

Your solution will be of the form:

$$x < a \text{ and } x \geq b.$$

4  
3  
2  
1

Your solution will be of the form:

$$a \leq x < b.$$

# LINEAR RELATIONSHIPS

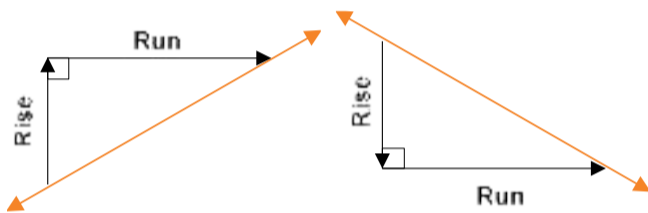
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## 07 GRADIENTS OF STRAIGHT LINES

### Gradient or steepness

- The **gradient** ( $m$ ) of a line is a measure of its **steepness**.
  - The gradient measures how far a line rises or falls within a given horizontal distance between two points on the line.
  - Gradient can be indicated by a positive or negative number. The **larger the gradient**, the steeper the line.
- The gradient is found by measuring the ratio of the vertical "rise" to the horizontal "run", as seen in the diagrams below.

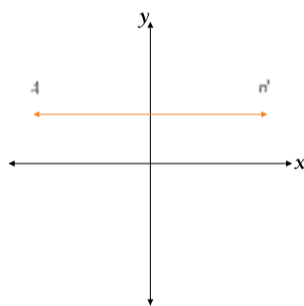
$$\text{Gradient} = m = \frac{\text{Rise}}{\text{Run}}$$



The line rises so the gradient is **positive**.

The line falls so the gradient is **negative**.

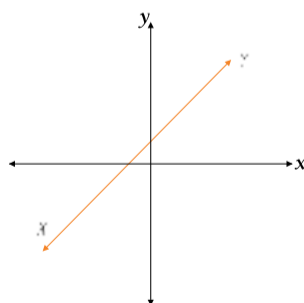
- Gradients of various straight lines are outlined below.



$AB$  is a horizontal line with no steepness.

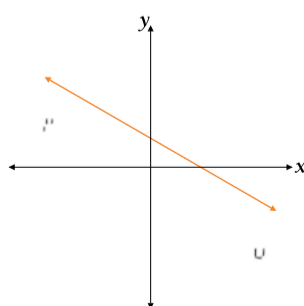
$$m = \frac{\text{rise}}{\text{run}} = \frac{0}{\text{run}} = 0$$

Hence, gradient is zero,  
 $m = 0$



$XY$  rises as we move from left to right.

Hence we say its **gradient is positive**,  
 $m > 0$

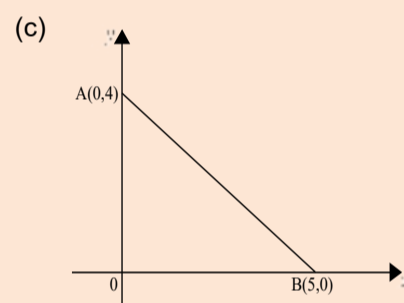
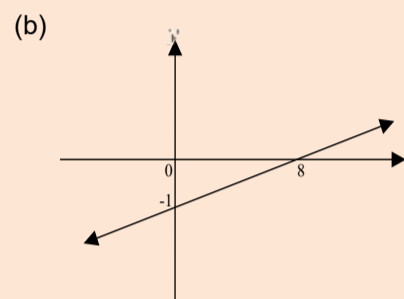
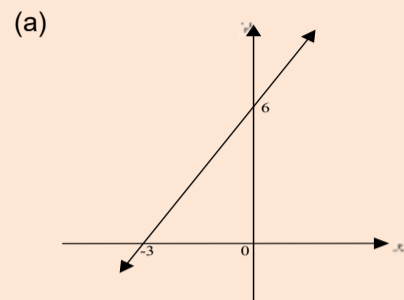


$PQ$  falls as we move from left to right.

Hence we say its **gradient is negative**,  
 $m < 0$

### Example

Find the gradient of the each of the lines shown in the following diagrams:



### Solution

- (a) The **line rises** as we move from left to right. Hence the **gradient is positive**. Find the ratio of "rise" and "run".

$$\begin{aligned} \text{Gradient} &= \frac{\text{Rise}}{\text{Run}} \\ &= \frac{6}{3} \\ &= 2 \end{aligned}$$

- (b) The **line rises** as we move from left to right. Hence the **gradient is positive**.

$$\begin{aligned} \text{Gradient} &= \frac{\text{Rise}}{\text{Run}} \\ &= \frac{1}{8} \end{aligned}$$

- (c) The **line AB falls** as we move from left to right. Hence the **gradient should be negative**.

$$\begin{aligned} \text{Gradient} &= \frac{\text{Rise}}{\text{Run}} \\ &= -\frac{4}{5} \end{aligned}$$

### Note to Student:

Gradients are usually expressed as simplified proper or improper fractions rather than as a decimal.