

YEAR 9 | NUMBER AND ALGEBRA

VOL.1 | ALGEBRAIC TECHNIQUES &
SURDS AND INDICES

YEAR

9

MATHS

MAX SERIES™



THE HSC EXPERTS

03 CONSTRUCTING EQUATIONS

Using equations to solve word problems

- Word problems can be categorised into:
 - (i) Number Problems
 - (ii) Measurement Problems
 - (iii) Money Problems
 - (iv) Age Problems
- To solve word problems:
 - Step 1:** Underline the keywords that suggest mathematical operations.
 - Step 2:** Select a pronumeral to represent the unknown quantity.
 - Step 3:** Translate the word problem into an equation.
 - Step 4:** Solve the equation.
 - Step 5:** Check that the solution satisfies the given problem.
 - Step 6:** State your answer in words.
- The table below shows keywords for addition and subtraction.

Word	Operation	Word	Operation
Plus	+	Minus	-
Sum	+	Difference	-
More than	+	Less than	-
Increased by	+	Decreased by	-
Gain	+	Loss	-

- The table below shows keywords for multiplication and division.

Word	Operation	Word	Operation
Times	×	Ratio	÷
Multiplied by	×	Divided by	÷
Product of	×	Quotient of	÷
Twice or double	×2	Half	÷2
Triple	×3	One-third	÷3

- In measurement and money problems, don't forget to include units in your answer.

Number problems

- In number problems, you are required to find an unknown number given its relationship with the other number(s).
- Number problems could involve
 - (i) Two numbers
 - (ii) Two or three consecutive numbers
 - (iii) Two of three consecutive even or odd numbers

Example 1

The sum of two consecutive numbers is 87 .
Find the numbers.

Solution

Step 1: Underline the keywords that suggest mathematical operations.

Step 2: Select a pronumeral to represent the unknown quantity.

Let the two consecutive numbers be x and $x + 1$.

Step 3: Translate the word problem into an equation.

$$\text{The sum of two consecutive numbers} = 87$$

$$x + (x + 1) = 87$$

Step 4: Solve the equation.

$$x + (x + 1) = 87$$

$$2x + 1 = 87$$

$$2x = 86$$

$$x = 43$$

Step 5: Check that the solution satisfies the given problem.

$$x = 43$$

$$x + 1 = 43 + 1 = 44$$

$$43 + 44 = 87$$

Step 6: State your answer in words.

The two consecutive numbers are 43 and 44.

Example 2

The sum of two consecutive even numbers is 146 .
Find the numbers.

Solution

Step 1: Underline the keywords that suggest mathematical operations.

Step 2: Select a pronumeral to represent the unknown quantity.

Let the two consecutive even numbers be x and $x + 2$.

Step 3: Translate the word problem into an equation.

$$\text{The sum of two consecutive even numbers} = 146$$

$$x + (x + 2) = 146$$

Step 4: Solve the equation.

$$2x + 2 = 146$$

$$2x = 144$$

$$x = 72$$

Step 5: Check that the solution satisfies the given problem.

$$\text{The smaller even number: } x = 72$$

$$\text{The next consecutive even number: } x + 2 = 74 .$$

Step 6 : State your answer in words.

The two consecutive numbers are 72 and 74 .

03 CONSTRUCTING EQUATIONS (MEASUREMENT PROBLEMS)

11. A water storage tank is two-thirds full. After 500 litres of water is drained out, the tank is two-fifths full.
- (a) Let the full capacity of the water storage tank be x litres. Write an expression in x for the initial amount of water in the tank.
- (b) Write an equation and solve it to find the capacity of the water storage tank.

12. A truck is loaded with 80 drums. Each drum weighs either 45 kg or 30 kg. The total mass of the 80 drums is 2970 kg.
- (a) Let the number of drums weighing 45kg be x . Write an expression in x for the total mass of 45kg drums and for the total mass of 30 kg drums.
- (b) Form an equation and solve it to find the number of drums weighing 45 kg.

LEVEL

4
3
2
1

NOTES TO STUDENTS

The amount of water in the tank before and after draining will be a fraction of x .

Total weight =
number of drums \times weight of drums

The question only asks for the number of 45kg drums (x).

Solving and graphing inequalities of the form $a \leq x \leq b$

- The inequality $a \leq x \leq b$ means that x is between a and b .

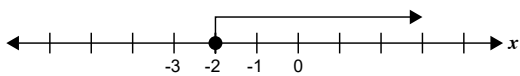
$$a \leq x \leq b$$

$$x \geq a \quad \text{AND} \quad x \leq b$$

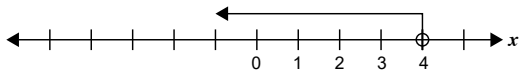
- Therefore, the inequality of the form $a \leq x \leq b$ can be expressed as $x \geq a$ AND $x \leq b$.
- Some examples of inequalities of this form are listed below.
 - $-3 \leq x < 4$
 - $3 \leq 2x < 7$
 - $-3 \leq x - 1 \leq 5$
- Let's graph the inequality $-3 \leq x < 4$.

– This means that $x \geq -3$ AND $x < 4$ at the same time.

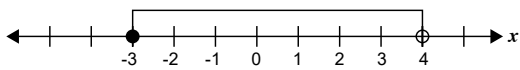
– The graph of $x \geq -3$ is:



– The graph of $x < 4$ is:



– The inequality $-3 \leq x < 4$ is the intersection of the above two graphs:



Example 1

Solve the following inequality and graph the solution on a number line.

$$4 \leq 2x < 10$$

Solution

Step 1: Separate the inequality into the two simultaneous statements.

$$4 \leq 2x \quad \text{AND} \quad 2x < 10$$

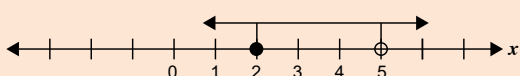
Step 2: Solve each inequality as usual.

$$4 \leq 2x \qquad 2x < 10$$

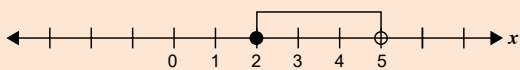
$$\frac{4}{2} \leq x \qquad x < \frac{10}{2}$$

$$x \leq 2 \qquad x < 5$$

Step 3: Graph each solution on the number line.



The solution is the region where the two lines overlap.



$$2 \leq x < 5$$

Example 2

Solve the following inequality and graph the solution on a number line.

$$3 < 2x + 1 \leq 7$$

Solution

Method 1

Step 1: Separate the inequality into the two simultaneous statements.

$$3 < 2x + 1 \quad \text{and} \quad 2x + 1 \leq 7$$

Step 2: Solve each inequality as usual.

$$3 < 2x + 1 \qquad 2x + 1 \leq 7$$

$$3 - 1 < 2x \qquad 2x \leq 7 - 1$$

$$2 < 2x \qquad 2x \leq 6$$

$$\frac{2}{2} < \frac{2x}{2} \qquad \frac{2x}{2} \leq \frac{6}{2}$$

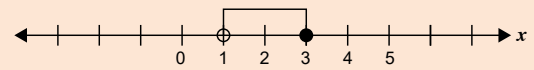
$$1 < x \qquad x \leq 3$$

This means x must be “greater than 1” and “less than 3” at the same time.

Step 3: Graph each solution on the number line.



The solution is the region where the two lines overlap.



This may be rewritten as one expression in the form $a < x \leq b$.

$$\therefore 1 < x \leq 3$$

Method 2

Step 1: Solve the inequality as it is written.

$$3 < 2x + 1 \leq 7$$

Treat this like any other inequality problem; but when transposing terms it is important to apply inverse operations to all sides of the inequality.

Step 2: Solve as normal making sure to balance both sides of the inequality.

$$3 < 2x + 1 \leq 7$$

$$3 - 1 < 2x + 1 - 1 \leq 7 - 1$$

$$2 < 2x \leq 6$$

$$\frac{2}{2} < \frac{2x}{2} \leq \frac{6}{2}$$

$$1 < x \leq 3$$

Note to students:

x is between 1 and 3 with 3 inclusive.

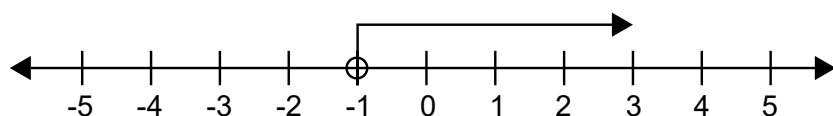
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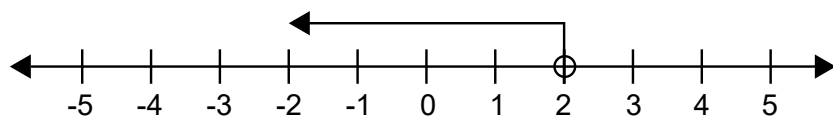
04 INEQUALITIES (REPRESENTING LINEAR INEQUALITIES)

2. Write inequality statements expressed by the following number lines.

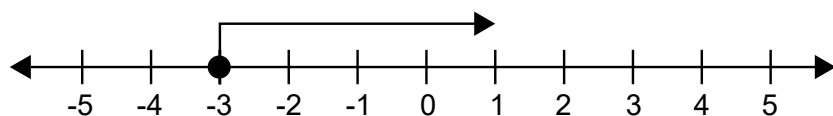
(a)



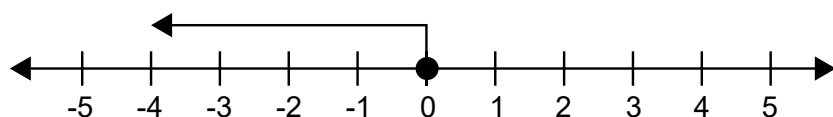
(b)



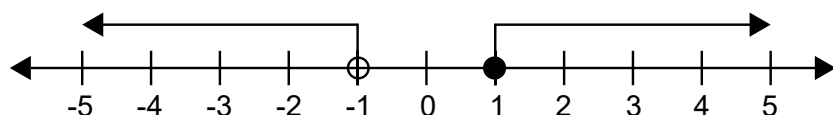
(c)



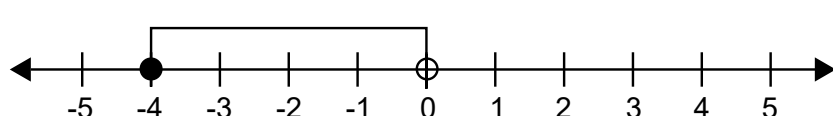
(d)



(e)



(f)



LEVEL

4
3
2
1

NOTES TO STUDENTS

Use the arrow head to help you remember which inequality sign to use.
An arrow pointing to the right \rightarrow means $>$.
An arrow pointing to the left \leftarrow means $<$.

4
3
2
1

Your solution will be of the form:
 $x < a$ and $x \geq b$.

4
3
2
1

Your solution will be of the form:
 $a \leq x < b$.

LINEAR RELATIONSHIPS

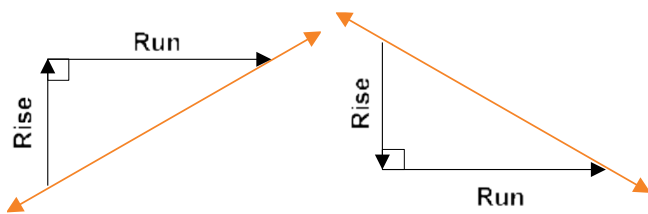
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07 GRADIENTS OF STRAIGHT LINES

Gradient or steepness

- The **gradient** (m) of a line is a measure of its **steepness**.
 - The gradient measures how far a line rises or falls within a given horizontal distance between two points on the line.
 - Gradient can be indicated by a positive or negative number. The **larger the gradient**, the steeper the line.
- The gradient is found by measuring the ratio of the vertical “rise” to the horizontal “run”, as seen in the diagrams below.

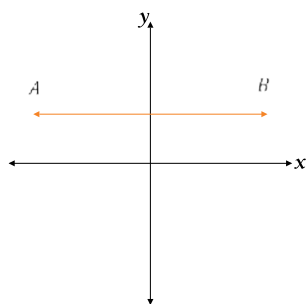
$$\text{Gradient} = m = \frac{\text{Rise}}{\text{Run}}$$



The line rises so the gradient is **positive**.

The line falls so the gradient is **negative**.

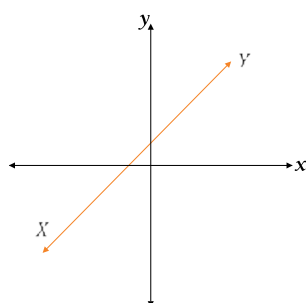
- Gradients of various straight lines are outlined below.



AB is a horizontal line with no steepness.

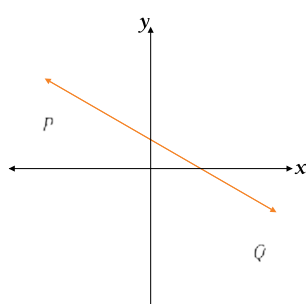
$$m = \frac{\text{rise}}{\text{run}} = \frac{0}{\text{run}} = 0$$

Hence, gradient is zero,
 $m = 0$



XY rises as we move from left to right.

Hence we say its **gradient is positive**,
 $m > 0$

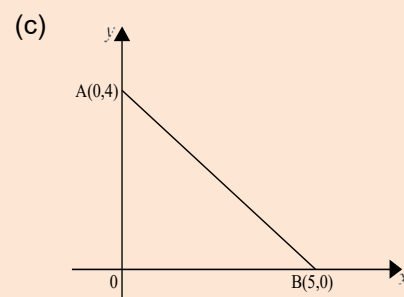
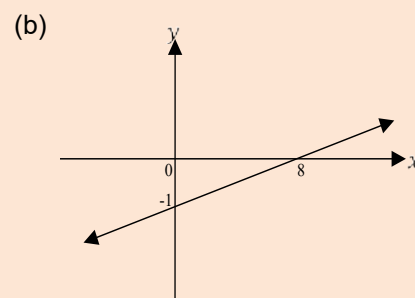
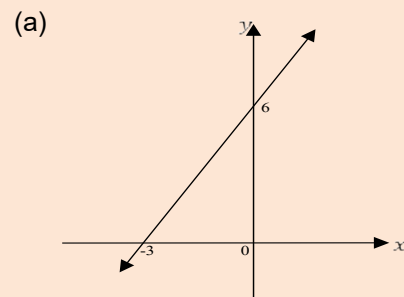


PQ falls as we move from left to right.

Hence we say its **gradient is negative**,
 $m < 0$

Example

Find the gradient of the each of the lines shown in the following diagrams:



Solution

- (a) The **line rises** as we move from left to right. Hence the **gradient is positive**. Find the ratio of “rise” and “run”.

$$\begin{aligned} \text{Gradient} &= \frac{\text{Rise}}{\text{Run}} \\ &= \frac{6}{3} \\ &= 2 \end{aligned}$$

- (b) The **line rises** as we move from left to right. Hence the **gradient is positive**.

$$\begin{aligned} \text{Gradient} &= \frac{\text{Rise}}{\text{Run}} \\ &= \frac{1}{8} \end{aligned}$$

- (c) The **line AB falls** as we move from left to right. Hence the **gradient should be negative**.

$$\begin{aligned} \text{Gradient} &= \frac{\text{Rise}}{\text{Run}} \\ &= -\frac{4}{5} \end{aligned}$$

Note to Student:

Gradients are usually expressed as simplified proper or improper fractions rather than as a decimal.